UNIVERSITY OF CAPE TOWN

FACULTY OF ENGINEERING AND THE BUILT ENVIRONMENT

AN ASSESSMENT OF THE CONTRIBUTION OF A SATELLITE LASER RANGING STATION AT MATJIESFONTEIN, WESTERN CAPE, SOUTH AFRICA: A SIMULATION STUDY USING LAGEOS AND ETALON DATA

Thesis prepared in partial fulfilment of the requirements for the degree

Bachelor of Science in Geomatics (BSc Geomatics)

By

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15 OCTOBER 2014
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ABSTRACT

Due to a planned relocation of the HartRAO Space Geodesy Program to Matjiesfontein, Western Cape South Africa, a study was carried out in order to assess the contribution of the planned SLR station via a simulation of SLR data from a station situated at Matjiesfontein for the year 2011. Being the unique parameter of SLR, the geocentre was chosen to be the key parameter during this assessment by estimating the dynamic geocentre via degree 1 spherical harmonic coefficients of the Earths’ gravity field in weekly arcs. For this purpose, three different solutions were estimated using SLR ranges from 02 Jan 2011 to 25 Dec 2011 to LAGEOS 1 & 2 and ETALON 1 & 2 satellites. The first of these solutions contained the current existing SLR network termed the baseline solution. These solutions were validated by comparison with the ILRSA geometric geocentre solutions. The second solution contained the simulated data. In the third solution the SLR station at Hartebeesthoek was excluded from the estimation to assess the influence on the resulting uncertainties.

During the estimation process, the core network was constrained meaning that only non-core station coordinates were estimated. For the baseline solution, the mean and RMS of the geocentre coordinates were estimated to be -0.1 and 3.6, -4.3 and 5.1, -0.6 and 9.2 mm in the X, Y and Z directions respectively. As a quality measure, the weekly uncertainties were estimated together with the geocentre estimates. The mean and RMS of uncertainties were estimated to be 0.7 and 0.7 mm, 0.6 and 0.7 mm, 1.9 and 1.9 mm in the X, Y and Z directions. These estimates indicate sub-millimetre accuracy in the dynamic geocentre estimation process and they represent the baseline solution. These directly translate to obtainable accuracies with data utilised for this study. The simulated data for the station Matj9999 was added in the baseline solution and the dynamic geocentre was re-estimated. The mean and RMS of these estimates were -0.6 and 3.2 mm, -3.4 and 4.2 mm, -1.1 and 8.5 mm in the X, Y and Z directions respectively. The mean and RMS of uncertainties for the X and Y components were both 0.6 mm and 1.8 mm for the Z component. This implies an improvement of 0.1 mm in the uncertainties compared to the baseline solution. In the third solution, the station at Hartebeesthoek was removed from the network and the mean and RMS of the geocentre estimates were -0.7 and 3.3 mm, -3.0 and 4.2 mm, -1.0 and 10.4 mm in X, Y and Z components. The mean and RMS obtained from the third solution was 0.7 and 0.7 mm, 0.7 and 0.7 mm, 2.0 and 2.2 mm in X, Y and Z directions. These solutions show an improvement of 0.1 mm in all components when the simulated data is added to the network.
DEDICATION

This thesis is dedicated to my family with special thanks to Mr. Simon Tembile Bantom, Mrs. Lulama Bantom, Mr. Lwando Bantom and Mrs. Mandisa Mlonzi for their continued support and encouragement in the duration of my studies. What all of you did for me will never be forgotten.
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LIST OF ACRONYMS

HartRAO – Hartebeesthoek Radio Astronomy Observatory (SA)
NRF – National Research Foundation (SA)
SLR – Satellite Laser Ranging
LLR – Lunar Laser Ranging
GNSS – Global Navigation Satellite System
DORIS – Doppler Orbitography and Radio Positioning Integrated by Satellite
VLBI – Very Long Baseline Interferometry
LAGEOS - Laser Geodynamics Satellites
LEO – Low Earth Orbits
ILRS – International Laser Ranging Services
IGS – International GNSS Services
IDS – International DORIS Services
IVS – International VLBI Services
IERS – International Earth Rotation Services
ITRF – International Terrestrial Reference Frame
ITRS – International Terrestrial Reference System
ICRS – International Celestial Reference System
ICRF – International Celestial Reference Frame
AWG – Analysis Working Group
GGOS – Global Geodetic Observing System
GSFC – Goddard Space Flight Centre (USA)
IAG – International Association of Geodesy
NASA – National Aeronautics and Space Administration (USA)
EOP – Earth Orientation Parameters
UT1 – Universal Coordinated Time
SINEX – Solution independent Exchange Format
CDDIS – Crustal Dynamics and Data Information System (NASA)
ILRSA– ILRS Combination Centre for Final Products
An Assessment of the Contribution of a Satellite Laser Ranging Station at Matjiesfontein, Western Cape, South Africa: A Simulation Study.

1. INTRODUCTION AND OVERVIEW

The current global network for Satellite Laser Ranging consists of about 50 active stations, 26 inactive stations and 5 engineering stations, and many of the active stations are co-located with other space geodesy techniques such as GPS, VLBI and DORIS whilst some sites are co-located with lunar laser ranging (LLR). The distribution of SLR stations is depicted in Figure 1. Apparent from Figure 1 is the uneven distribution of SLR stations in the global network. The network has more stations situated in the northern hemisphere compared to the number of stations located in the southern hemisphere. Combrinck (2010) points out that this weak distribution causes geometrical and coverage weaknesses in the global SLR network. The direct implication of this uneven distribution of SLR stations is that the uncertainties with which the location and motion of the geocentre can be determined are higher than they would have been if the global network had stations distributed evenly as per the DORIS network throughout both hemispheres, as pointed out in literature (Kuzin & Tatevian, 2006; Dong et al., 1998).

Since its inception in 1964 by NASA (Degnan, 1994), SLR has contributed to the global terrestrial reference frame (ITRF) definition which is internationally recognized as the realization of the international terrestrial reference system (ITRS). A reliable reference frame is an important aspect with respect to geodynamics and monitoring physical Earth processes such as crustal motion, glacial rebound, hydrological processes and sea level rise to mention a few, due to the fact that these long term physical processes need to be determined with high accuracies and precisions. The contribution of SLR to the understanding these Earth processes is part of the objectives set out in the Global Geodetic Observing System (GGOS) and is clearly outlined by Plag et al., (2009).

It is thus important to maintain and update the ITRF in order to further allow for definitions and integrations of various national datums such as South Africa’s Hart94 datum as these utilise one or two of the ITRF positions as their origin. The ITRF is defined by a set of combined coordinates of the global network from the four space geodesy techniques. During the ITRF definitions, the computational burden caused by nuisance parameters is overcome.
by the determination of intra-technique (SLR, GNSS, VLBI, DORIS) weekly solutions at analysis centres and these solutions are then later combined via inter-technique combinations by utilizing tie vectors at co-located sites (Altamini et al., 2011).

With respect to the ITRF, the International Laser Ranging Service (ILRS) network is dedicated in the determination of the Earth’s centre of mass (CoM)/geocentre as its unique parameter by estimating the translation parameters from a 7 parameter transformation or from estimating the degree one harmonic coefficients of the Earth’s geopotential as explained in Govind et al. (2010). SLR is the only technique from which geocentric station positions can be determined and as such, it is only technique from which the geocentre can be computed directly (Steinberger et al., 2010). Altamini et al., (2002) showed that a multi-technique combination of geocentre results led to the dilution of the precision of these results, having compared SLR and GPS solutions within the same study, SLR yielded the most consistent results. This justifies the definition of the geocentre from the SLR technique. The SLR network also plays a role in the geometric determination of the ITRF scale together with the VLBI global network.

Figure 1: Locations and distribution of SLR stations that make up the global network (status 2011). Source: ILRS (2013)
The SLR technique cannot directly determine UT1 due to the coupling of variations in UT1 and the variations in the satellite’s orbital node caused by unmodeled perturbations of the satellite, however, the UT1 time rate of change and length of day (LoD) can be determined from SLR observations (Plag et al., 2009).

In addition, according to the Report of the 1994 Belmont Workshop, SLR measurements are also dedicated in the:

- Detection and monitoring of tectonic plate motion, crustal deformation, Earth rotation and polar motion.
- Modelling of the spatial and temporal variations of the Earth’s gravity field.
- Determination of Earth and ocean tides.
- Monitoring of variations in the centre of mass of the total Earth system via degree one geopotential coefficients.
- Detection and monitoring of vertical motions produced by post-glacial rebound, subsidence, and atmosphere and ocean loading.

In its early years, the SLR technique had reported precisions of more than a few metres. However, currently this technique is able to obtain precisions better than a few millimetres through better modelling of systematic effects, addition of SLR stations in the core network, and the advancement of the SLR technology and capabilities throughout the years. However, given the historical improvements in precision and technology, SLR ranging precisions continue to be burdened by the capabilities of the ground stations (Merkowitz et al., 2012).

This improvement is also attributable to the hard and efficient work of the Combination Centres (CC’s) and the intra-technique Analyses Working Groups (AWG) in ensuring that the results submitted to the technique centres for SLR are of superior quality. It is also the continuous work of the governing bodies and societies such as ILRS, IAG, GGOS, and IERS in improving and imposing computation standards and data baselines as minimum requirements for both current and prospective SLR sites around the world in order to control the quality and quantity of obtainable data.

The ability to control the quality and quantity of the data contributed by each station is of importance because this will determine which stations will contribute valuable data for various scientific products such as those mentioned above. According to Govind (1997), there are various factors that determine the quantity of data that can be contributed by an SLR
station which include but are not limited to weather and cloud cover, satellite elevation/availability/visibility, work practices (operation hours), length of shifts and periodic station maintenance.

In order to control the quality and quantity of SLR data, the ILRS publishes a list of annual guidelines to be used by the analysis working groups (AWG). For low Earth satellite (LEO) the data quantity is set at 1000 passes p/a, 400 passes p/a for LAGEOS-1/2 and an annual 100 passes for high pass satellites setting the baseline at 1500 passes p/a. The quality guidelines on the other hand are stated as 1cm normal point precision, 2cm short term bias stability and 1cm long term bias stability. The guidelines for operational compliance are as follows: delay of 12 hours for data delivery at a specified ILRS normal point file format submitted with a site log. The best operating stations in the global network are at Yarragadee and Mt Stromlo in Australia and Zimmerswald, Switzerland including the Changchun station in China.

The standards and requirements outlined above continue to increase and intensify due to the need for high quality products, and the ability of each station in the network to keep up with these accuracy and performance standards will be influenced by the state of the instrumentation and the level of environmental impacts from the locality of the tracking station such as the severity of polluting factors e.g. air pollution, unfavourable weather and climate, unstable geology etc. in the case of SLR as it utilises the optical wavelengths.

Due to the accuracy and performance demands outlined above, and the fact that the HartRAO Space Geodesy Program operates in close proximity to large cities (Johannesburg and Pretoria) and industrial areas which are a source of these polluting factors, the HartRAO Space Geodesy Program is planning to relocate its activities to a new site in Matjiesfontein Western Cape, South Africa (Nickola, 2012). This site is one three sites which were proposed amongst Lesotho and Sutherland, South Africa. For the sake of completeness it is thus sufficient to highlight the performance and capabilities of the SLR station in HartRAO.

1.1 BACKGROUND TO STUDY

The HartRAO Space Geodesy Program operates under the auspices of the South African National Research Foundation (NRF). This station has been operating in the region since it was officially introduced in 2000 (HartRAO, 2000). However, there was a temporary SLR station in Sutherland which was later moved to HartRAO but was never used for space
geodesy purposes. The station at HartRAO belongs to NASA’s MOBLAS systems and is called MOBLAS-6 (MOBile LASer Ranger-6). The operation and assembly of the MOBLAS-6 system is discussed in Combrinck (2010). The station was provided by NASA in partnership with the NRF. The HartRAO MOBLAS-6 station is a 5Hz system and averages approximately 66 data points per normal point for LAGEOS satellites (Combrinck, 2010).

The contribution of this station over the period from 2011 to the end of 2012 is depicted in plots below in Figures 2 – 5.

The objective of the plots in Figures 2 – 5 is to compare MOBLAS-6 (Hart7501) station at Hartebeesthoek with the current highest data yielding stations at Yarragadee (Yarr7090) and Zimmerswald (Zimm7810) on the basis of the quantity of observations and quantity of observed passes for both LAGEOS and ETALON satellites. The idea is to highlight the contribution of the MOBLAS-6 station to the data used for this study amongst other stations as it is recognised as a core site. These plots were generated from net weekly observations contributed by each station.

Figure 2: Number of observations for ETALON-1 (left) and number of observed passes of ETALON-1 (right) satellite between 2011 and 2012.

1 The data used to generate these plots was extracted from the report cards personally generated by Dr Ramesh Govind (UCT) as a member of the ILRS AWG for stations that contributed data to the ILRS official weekly products.
Figure 3: Number of observations for ETALON-2 (left) and number of observed passes of ETALON-2 (right) satellite between 2011 and 2012.

Figure 4: Number of observations for LAGEOS-1 (left) and number of observed passes of LAGEOS-1 (right) satellite between 2011 and 2012.

Figure 5: Number of observations for LAGEOS-2 (left) and number of observed passes of LAGEOS-2 (right) satellite between 2011 and 2012.
From figures 2-5, it is clear that the data quantity and the number of passes of LAGEOS satellites are higher than that of ETALON satellite for all three stations. An explanation for this is that LAGEOS satellites have more frequent passes than ETALON satellites with orbital periods of 225 minutes and 676 minutes for LAGEOS and ETALON satellites respectively. This is because ETALON satellites orbit at higher altitudes compared to LAGEOS satellites and this poses difficulties for ranging sites around the world thus ETALON’s less data yield (Appleby, 1998). However, a further influence to these distributions is operational preferences for the agencies that are in charge of these respective stations, such that both quantities for passes and observations are overall favoured by satellite’s orbital period and the dedication of the operating team.

The proposed Space Geodesy Outstation in Matjiesfontein is a project by the Space Geodesy Program based at HartRAO. Matjiesfontein is a town situated along on the N1 national route located in the semi-arid region of the Karoo roughly two and a half hours from Cape Town CBD. The proposed space geodesy site is located approximately 5km south of the town Matjiesfontein. The locality of this site is depicted in Figure 6 below. The chosen location is to host all four space geodesy techniques (VLBI, DORIS, SLR and GPS) and amongst these, Lunar Laser Ranging (LLR) is proposed to be hosted at Matjiesfontein.

Key factors considered in assessing Matjiesfontein’s suitability to host a space geodesy program where clear skies, low horizon, minimum noise and access to basic resources. Amongst these factors, rainfall was also assessed in the proposed region and was found to be low compared to the currently existing site at HartRAO in Pretoria (Fourie et al., 2007). The implication of this is that observation time for SLR will be increased and thus the data volume will increase as well. An important climatological element for SLR is essentially the number of cloud free days as this would affect ranging at any SLR site due to the use of optical wavelengths. This means that the prospective location would yield more annual data quantity compared to the existing site.
The rationale for this study is therefore to investigate the contribution of the proposed SLR station at Matjiesfontein to the global ILRS network. The next section outlines the objectives of this study.

### 1.2 OBJECTIVE OF THE STUDY

The main objective of this study is assessing the contribution of the new SLR station at Matjiesfontein, South Africa. This contribution refers to the improvement in the level of uncertainties of the weekly geocentre estimates when the new SLR station is operational compared to the weekly solutions obtained from the existing global network. Further, to investigate whether or not it will be suitable to keep the MOBLAS-6 SLR station at HartRAO operational should Matjiesfontein be the ultimate chosen location. A method for doing so is analysing the improvement in geocentre estimates from the combinations where MOBLAS-6 is included, and excluded in the geocentre estimation process followed in this study. If the uncertainties improve both with and without MOBLAS-6 then perhaps all space geodesy activities should be migrated to the new site in order to avoid maintenance costs.
According to Govind (2006), the best way to assess this contribution is via a simulation that will systematically create observations for the proposed non-existing station which will then be augmented with the global network to compute new weekly arc geocentre estimates. Both the analyses strategies and the method for normal point simulation are detailed in chapter 4.

A similar study was carried out by Govind (2006) in Australia, in which it was found that an additional SLR station in Australia would contribute towards the improvement in the geocentre determination by as much as 7 mm in the X direction, -3 mm in the Y direction and 13 mm in the Z direction. The theory behind SLR dynamic geocentre determination is covered in chapter 3. To date, only a few simulation studies are available for review. However many of these are institutional private studies which are never published for academic use thus it is difficult to get access to the resulting reports.

1.3 SPACE GEODESY TECHNIQUES

Space geodesy as opposed to terrestrial geodetic methods utilizes space borne artificial satellites that accommodate various scientific instruments designated for various geodetic missions. Examples of these missions include satellite to satellite tracking (SST), determination of the Earth’s geopotential, gravity gradiometry, satellite altimetry, various GPS missions etc. Some of the valuable products of space geodesy from these missions include Precise Orbit Determination (POD), satellite solutions to gravity field models, determination of station positions and their velocities, lunar measurements, determination of EOPs and their time variations, time, geodynamics, reference frame definitions both celestial and terrestrial and the transformation parameters between celestial and terrestrial reference frames.

However satellite orbit determination is arguably the most important scientific product of space geodesy as it is the fundamental for all space geodesy techniques except VLBI as it doesn’t utilize artificial satellites. These products in turn are utilized by various scientific communities and agencies to analyse processes such as sea level rise and absolute sea level monitoring, obtaining sea surface topography, glacial rebound, oceanic and atmospheric circulation, climate change and tectonic processes (Nickola, 2012). Currently there are five major techniques utilized in space geodesy which are SLR, DORIS, VLBI, GPS and the
trending LLR. These techniques are detailed in the sections that follow with DORIS, VLBI, GPS, and LLR given in short summary form by way of consolidation.

1.3.1 SATELLITE LASER RANGING (SLR)

Range measurements are carried out to space borne satellites equipped with corner cube reflectors of known size and shape. A typical mobile laser ranger is shown in Figure 6. This particular example is MOBLAS-6 at HartRAO, however most systems in the global network are permanent. A high energy laser pulse is directed to the satellite and the two-way time-of-flight of the laser pulse is measured. Once the laser is transmitted, the gate of a precise timing device at the transmitter is opened and the reflected pulse is received at a photomultiplier co-located with the laser. This two-way time-of-flight is used to determine the round trip distance of the pulse which is then halved to obtain the range from the ground station to the satellite. After systematic error sources have been accounted for, the range $(R_t)$ equation for this technique is as follows:

$$R_t = \frac{1}{2} \Delta t \cdot c$$

In the equation above, $t$ is the round trip time-of-flight and $c$ is the speed of light in a vacuum. The main observable in this equation is the two way travel time. The SLR system is highly accurate compared to other space geodesy techniques due to the favourable propagation of laser light in the atmosphere (Torge & Muller, 2012). A modern SLR station is capable of achieving a normal point precision at the 1-3 mm level for a typical geodetic satellite with an accuracy of between 8-18 mm (Combrinck, 2010). These quality indicators will vary for different stations around the world due to available technology and technological advancements. Due to this high accuracy and the unambiguous observation of satellite orbits, SLR has proved useful in fine tuning GPS orbit solutions and in the calibration of satellite altimetry missions (Luthcke et al., 2002). With respect to satellite altimetry, SLR also provides high quality orbits for the determination of accurate heights for the definition of sea surface topography (SST) from which precise gravitational anomalies and geoidal heights can be determined. The contributions of SLR to POD are extensively outlined in Luthcke et al., (2002). The SLR system has only two segments: the space segment which consists of the
satellites equipped with corner cube reflectors and the ground segment which consists of the SLR ground network shown in Figure 1. These are both detailed in chapter 2.

During each satellite pass, the volume of data obtained will be determined by the frequency of the pulsed laser. The observations are then termed full rate/ single shot data which can be in the order of thousands of observations per second for the current KHz systems; however, there are stations that are capable of obtaining twice this order of magnitude in terms of data volume (2 KHz systems). Full rate data is archived at the relevant data centres (CDDIS) and utilized for further specialized geodetic studies. Due to this high volume of data, the full rate data is then compressed into normal point data at suitable/chosen epochs at ILRS recommended data points per normal point. In fact the range determined from the normal points is referred to as normal point range calculated as follows at a specific epoch in metres:

\[ NPR_t = \left( NP_{tof} \times c \right)/2 \]

The normal point time-of-flight reported in seconds and \( c \) is the propagation speed of light and is defined to be 299792458 m/s (Petit & Luzum, 2010). This range is then corrected for various systematic effects including atmospheric delay, satellite’s centre of mass (CoM), station bias and relativistic effects (Combrinck, 2010). The epoch in this case is chosen to be the instant the pulse left the telescope. During the parameter estimation process, the range reduced from time-of-flight measurements is subtracted from the calculated range to obtain the observed – computed (O – C) range residuals. The parameter estimation process is detailed in Combrinck (2010). The process involved in compressing full rate data into normal points is outlined in Seeber (2003).

Currently most SLR stations are manned, meaning that an operator has to move the telescope around to spot the satellite utilizing predicted orbits from the prediction centres. According to Combrinck (2010), the instrument will have pointing errors which emanate from mechanical tendencies such as misalignment, clock offsets, instrumental biases as well as orbital biases. To enhance station capabilities NASA is developing a prototype called the Next Generation Satellite Laser Ranging (NG-SLR or SLR2000) at the Goddard Space Flight Centre. The concept of SLR2000 was conceptually introduced by Degnan (1993). Some of the features of
this technology include full automation, eye safe, low cost, continuous 24 hour satellite tracking, and a normal point precision at the mm level, amongst others.

1.3.2 DOPPLER ORBITOGRAPHY and RADIO POSITIONING INTEGRATED by SATELLITE (DORIS)

DORIS was developed by the French Space Agency the Centre National d'Etudes Spatial (CNES) in partnership with Groupe de Recherché de Geodesie Spatiales (GRGS) and the Institut Geographique Nationale (IGN) (Plag et al., 2009). It is a satellite system dedicated to precise orbit determination and precise ground positioning. Other DORIS products include Length of Day (LoD), and polar motion which are also contributed to the IERS. UT1 cannot be independently determined from DORIS however UT1’s time rate of change can be estimated from DORIS data (Plag et al., 2009).

The system consists of ground beacons transmitting microwave signals on two frequencies (Dual frequency) namely 2036.25 MHz and 401.25 MHz. The dual frequency allows the system to remove ionospheric effects which affect the signal’s propagation in the Earth’s atmosphere. A diagram representing the DORIS system is shown in Figure 10. The space segment consists of satellites equipped with a passive receiver, a precise timing device for time tagging, and a download link to transmit the Doppler shifts measured at the satellite to the master control station in France as shown in Figure 10.

DORIS receivers are on board numerous satellites designated for other missions such as altimetry and remote sensing. DORIS’s uniqueness is with respect to the high densification and even distribution of its beacons around the world across both hemispheres, which is not the case with the other space geodesy techniques (Torge & Muller, 2012). This is due to deployment costs which are relatively low for DORIS compared to other techniques. One of these beacons currently exists and is operational at HartRAO and is co-located with other space geodesy techniques (GPS, SLR, and VLBI).

With respect to reference frame definitions the International DORIS Service (IDS), which is the administrative body for DORIS operations, delivers station positions and Earth orientation parameters in the form of weekly SINEX files (Plag et al., 2009). With reference to the ITRF08, the DORIS network contributed weekly SINEX files from seven Analysis Centres (AC’s) and from seven DORIS satellites including TOPEX/Poseidon, SPOT-2, SPOT-3,
Currently DORIS is capable of producing orbits with accuracies at the cm level in post processing whilst IDS provides tracking station coordinates and their velocities with accuracies better than 1cm (Torge & Muller, 2012). The organizational aspects of IDS and its data centres, DORIS constellation and tracking stations, and future developments of the IDS are all outlined in Willis et al. (2012).

1.3.3 GEODETIC VERY LONG BASELINE INTERFEROMETRY (VLBI)

Amongst the space geodesy techniques mentioned thus far, VLBI is the only technique that does not make use of artificial satellites. This means that VLBI is unaffected by the Earth’s gravity field and other perturbations such as third body, orbit modelling and mismodelling errors that are associated with artificial satellites (Seeber, 2003). This technique makes use of distant celestial bodies called quasars. Due to their large distances from Earth (extra galactic), these celestial bodies are treated as reference marks fixed in space. Quasars emit radio signals at multiple frequencies continuously.

In VLBI, two widely separated radio telescopes (at least 20m in diameter) are used to detect electromagnetic radiation from multiple quasars. The objective is to determine the baseline between these two stations that belong in the IVS global network. This is done via the determination of the time delay between the receptions of this EM radiation at the two stations. This is because the radiation emitted by the quasars is received in the form of parallel waveforms perpendicular to the direction to the quasar itself. This time delay is the primary observable from which the baseline is derived. The equation below summarizes this procedure.

\[
\tau (t) = \frac{1}{c} B.S = \frac{1}{c} [\Delta x \cos \delta_s \cos h_s + \Delta y \cos \delta_s \sin h_s + \Delta z \sin \delta_s]
\]

And

\[
h_s = GAST - \alpha_s
\]
Where $\Delta x$, $\Delta y$, $\Delta z$ are the components of the baseline (B), $\delta_s$, $\alpha_s$ are the spherical equatorial coordinates of the quasar (S) and $c$ is the light speed in a vacuum with $h_s$ as the Greenwich hour angle of the quasar (S) (Seeber, 2003). Corrections to this equation include atmospheric corrections (Ionosphere & troposphere), clock offsets at the stations and linear drifts.

In the equation above, the coordinates $\delta_s$ and $\alpha_s$ are in a celestial reference frame (ICRF); B is in terrestrial reference frame (ITRF) meaning that S is transformed into a terrestrial frame by utilizing EOP’s (Torge & Muller, 2012) which are treated as parameters in the estimation process and submitted as weekly products to relevant users. Measuring $\tau$ involves digitally recording the signals at the two stations with time tags. These are then electronically transmitted to one of seven IVS correlators where the two signals will be digitally correlated (IVS, 2010). Ionospheric compensation is done through dual-frequency observation of the S-band (2.3GHz) and X-band (8.4 GHz) (Torge & Muller, 2012) at each station.

VLBI only provides relative positions as apparent in the equation. This means that geocentric positions can only be obtained if one other station is known from another technique (SLR). VLBI observation arcs are designated as sessions with a single session spanning 24 hours and involving at least 8 IVS/GGOS stations observing to at least 60 quasars and the whole network is successively built by allowing for overlaps (Plag et al., 2009).

The uniqueness of VLBI is with respect to the monitoring and maintenance of the ICRF and its relationship to the ITRF by directly observing UT1 and the nutation parameters (Seeber, 2003). VLBI in conjunction with SLR also contributes to the scale of the ITRF (Altamini et al., 2011; Plag et al., 2009). Other VLBI products to the IERS include Time series of baseline lengths, tropospheric parameters, daily EOP through the transformation of S, and station positions (IVS, 2010).

The current network of stations provides station positions with an accuracy of 5mm and the proposed next generation system will improve the capabilities with position accuracies of 1mm and 0.1mm/year for velocities globally amongst other improvements.
1.3.4 GLOBAL NAVIGATION SATELLITE SYSTEM (GNSS)

This is the collective term used to identify a system of GPS, GLONASS and Galileo satellites. The name change from GPS to GNSS was decided by the IGS governing board with the intention of incorporating GLONASS and other future GNSS missions. This technique uses a microwave to carry pulsed signals generated at the satellite, together with a timing device; the one-way travel time is measured to derive the relative positions between a receiver on the ground and satellites in space. This one-way time is termed the time difference and is determined by timing a code generated at the satellite and cross correlating this code with a copy generated at the receiver. Due to relativistic effects, clocks on board the satellite are not synchronised with clocks on the receiver thus a clock synchronization error (clock offset) needs to be accounted for. The range derived therefrom is called the pseudo range and is used for navigation purposes. The equation is as follows:

\[ R = c \Delta t = \rho + c \delta t, \text{ where} \]

\[ \rho = \sqrt{(X_s - X_p)^2 + (Y_s - Y_p)^2 + (Z_s - Z_p)^2} \]

In this equation, \( c \) is the speed of light in a vacuum, \( \Delta t \) is the time difference, \( \delta t \) is the clock offset and \( \rho \) is the calculated range containing the geocentric coordinates of the satellites (subscript s) and receiver (subscript p) which are obtained in the WGS84 system (Torge & Muller, 2012). The idea is that at least three satellites, spread around the horizon, are required to determine the range, however a fourth satellite is required to determine the clock offset. Good geometry will allow users to obtain a good P-DOP (position dilution of precision).

For geodetic and surveying purposes, carrier phase measurements need to be considered as this offers better accuracies than those required for navigation purposes. This is done via a comparison of phases between the wave generated at the satellite (carrier) and the copy generated at the receiver (reference) as shown in the following equations:

\[ \Delta \varphi = \varphi_c - \varphi_r \]

And this phase difference is then linked to the required range as follows:
\[ \Delta \varphi = \frac{2\pi}{\lambda} \left( \rho + N\lambda + c. \delta t \right) \]

The quantities \( \rho \) and \( c. \delta t \) are defined same as before. \( \lambda \) is the wavelength of the wave trains and \( N \) is the integer ambiguity. The process of resolving for cycle ambiguities is outlined in Seeber (2003) and the error budget for GNSS is outlined in Torge & Muller (2012). In addition, theory dealing with the modelling of ionospheric effects as well as other atmospheric effects on microwave methods such GNSS is outlined in (Alizadeh et al., 2013).

The GPS/GNSS system components include the space segment which consists of a constellation of 30+ satellites in near-circular orbits with periods of up to 12 sidereal hours at an altitude of 20200 km. The other component is the ground segment which has the responsibility of managing the global network of ground stations and the space segment. The GPS signal structure includes the L1 (1575.42 MHz) and L2 (1227.6 MHz) carriers which are harmonics of the fundamental frequency (10.23 MHz). These two carriers are modulated by the C/A (coarse acquisition), P (Precise)/ (Y-code) codes and the NAV (navigation message).

Modernization of this signal structure included an introduction of the L5 carrier modulated by the CNAV (civilian navigation). Another element is the modulation of the M –code on the L1 carrier and the L2C (L2 civilian) carrier and CNAV modulated on the L2C carrier. The C/A and the P codes are called Pseudo Random Noise (PRN) and are available to civilians for navigation. The navigation message is required to evaluate pseudo ranges and is contained in the navigation message (NAV) and it also contains the satellite’s broadcast ephemeris with accuracies quoted at the metre level (Torge & Muller, 2012). Satellite specific information can be found in the navigation message.

The dense global IGS network offers a great advantage in the definition of the ITRF due to more co-location of IGS stations with other techniques, which allows for more tie vectors in reference frame definition. IGS contribution to the ITRF08 is covered in Altamini et al. (2011).

1.3.5 LUNAR LASER RANGING (LLR)

The principle behind this technique is similar to that of SLR, in that a range is measured using pulsed laser and the time-of-flight is measured however there are differences in the parameter estimation due to more influences mostly related to the interaction of the two bodies (Earth
and Moon). In this case the reflective surface is a set retro-reflector array on the lunar surface as shown in Figure 1.9. According to Nicola (2012), station positions and the location of the moon can be determined with sub-centimetre accuracy. This technique also allows for precise range determination between the Earth and the Moon. The available retro reflectors on the lunar surface were placed by 3 NASA missions, Apollo 11, 14 & 15 between 1967 and 1971 and these were accompanied by two French reflectors which were deployed by two Soviet automatic lunar missions (Seeber, 2003) as shown in Figure 9.

From the data derived therefrom useful information such as the Earth – Moon dynamics, Earth dynamics, Earth’s precession and tidal acceleration can be obtained through data analysis (Dickey et al., 1994). In deriving the range, two sets of coordinates are required: Those of the retro reflector on the lunar surface and those of the telescope here on Earth. The two sets of coordinates are then referred to a heliocentric coordinate system, termed the mean heliocentric. The complete model for LLR range and the parameters involved is discussed extensively in Muller & Nordtvedt (1998), and the parameters contributed and expected to be contributed by LLR are categorized by Seeber (2003) as:

- Global parameters of the Earth – Moon system
- Earth rotation and
- Gravitational physics and relativity.

Currently there are four lunar rangers globally at: Apache point Lunar Laser-ranging Operation (APOLLO) in New Mexico, the McDonald Laser Ranging System (MLRS) in Texas, the Observatoire de la Cote d’Azur (OCA) in France and the Matera Laser Ranging Observatory (MLRO) in Italy. An LLR station is also planned for the southern hemisphere and is planned to be located in Southern Africa and it will be the only LLR station in the southern hemisphere (HartRAO, 2011).
Figure 7: SLR (Source: HartRAO)

Figure 8: VLBI (Source: HartRAO)

Figure 9: Locations of RRA (source: NASA)

Figure 10: DORIS (source: CLS)
2. SLR SYSTEM COMPONENTS AND OBSERVATION MODELLING FOR GEODESY

2.1 COMPONENTS OF THE SLR SYSTEM

The purpose of this chapter is to detail as well as inform the reader about the system and components of the SLR system as currently managed and administered by the ILRS. More information on the SLR system and infrastructure is available on the ILRS website. As mentioned, the ILRS coordinates and mandates all procedures and functions concerning SLR data, whether it is SLR data transfer and processing standards, formats for data transfer, publishing of results, or standards to be utilised by its recognised analyses centres. The ILRS is also responsible for publications concerning the state of the infrastructure which are published every two years.

2.1.1 SPACE SEGMENT

This component of the SLR system consists of approximately 34 satellites as of 2012 and approximately 40 satellites as of 2013 in various orbits around the Earth. The increase in the number of satellites tracked by the ILRS is an indication and recognition of the quality of SLR data and its products. A small proportion of these satellites constitute the spherical geodetic satellite type satellites which orbit at near stable orbits around the Earth. Apart from these geodetic missions, other missions supported by the ILRS include remote sensing missions, navigation and experimental missions with different products different from orbit modelling and geodetic parameter estimation.

The pre-requisite for these satellites to be tracked by the ILRS is that they have to be equipped with corner cube reflectors or a retro reflector array of known size, shape and mass for the purposes of satellite CoM offset modelling. With respect to navigation (GPS & GLONASS) and other missions, SLR tracking mainly offers POD in assistance to these missions in obtaining high quality scientific products. Navigation constitutes a large proportion of satellites that are tracked via SLR, and the numbers will grow depending on the introduction of new GNSS missions. The ILRS’s Mission Working Group (MWG) is responsible for approving SLR missions as well as other missions and designating priorities to new missions for the purposes of tracking. The majority of these satellites were initially tracked by SLR in 2012 and some satellites were tracked earlier than that.
The geodetic type satellites include ETALON-1 & -2 and LAGEOS-1 &-2 satellites, the Japanese Ajisai, the French Stella & Starlette, and the German GFZ-1 satellites. The primary geodetic missions are the ETALON and LAGEOS missions with respective products utilized in global parameter estimation for reference frame and gravity field modelling. Primary products from these satellites include daily 7- day arc parameters (geocentric coordinates of global network, polar motion and LoD), and weekly 7 – day arc parameters (geocentric coordinates of global network, polar motion and LoD, orbit solutions) (Pavlis et al., 2010). Mission parameters for these are summarized in Table 2.1.

<table>
<thead>
<tr>
<th>SATELLITE</th>
<th>MASS (KG)</th>
<th>DIAMETER (CM)</th>
<th>PERIOD (MIN)</th>
<th>INCLINATION (DEG)</th>
<th>PERIGEE HT (KM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAGEOS-1</td>
<td>407</td>
<td>60</td>
<td>225</td>
<td>109.8</td>
<td>5.86</td>
</tr>
<tr>
<td>LAGEOS-2</td>
<td>405.4</td>
<td>60</td>
<td>222</td>
<td>53.6</td>
<td>5.62</td>
</tr>
<tr>
<td>ETALON-1</td>
<td>1415</td>
<td>129.4</td>
<td>676</td>
<td>64.8</td>
<td>19.12</td>
</tr>
<tr>
<td>ETALON-2</td>
<td>1415</td>
<td>129.4</td>
<td>675</td>
<td>64.8</td>
<td>19.12</td>
</tr>
</tbody>
</table>

With respect to satellite orbit predictions, the ILRS’s official prediction format is the consolidated prediction format (CPF) implemented as of 30 June 2006 and it replaces the previous tuned inter-range vectors (TIRV) format (ILRS, 2013). CPF offers high quality orbit predictions to the ILRS tracking complement in daily geocentric X, Y, and Z or geodetic coordinate tables for ILRS approved targets. Satellite predictions are published by 22 prediction centres around the world which work in conjunction with the ILRS. The predicted orbits are then compared and analysed daily at the NERC Space Geodesy Facility in East Sussex, England.

In non-geodetic type satellites, SLR corner cube reflectors are usually co-located with sensors utilised for other space geodesy techniques such as GPS and DORIS. This is referred to as satellite co-location. Satellite co-location is not utilized during ITRF definitions however its main application is in the validation of orbits determined from different techniques for the same satellite. Orbit validation is only possible if the sensors are co-located at the same satellite. The geodetic type satellites do not have co-location on board as these are purely
designed for ranging purposes. Satellite co-location is defined in the same manner as the site co-location in that all the sensors are linked via tie vectors defined pre-launch.

2.1.2 GROUND SEGMENT

The current state of the global network for which the ILRS is responsible for, consists the stations depicted in Figure 1.1, however there are several new proposals around the world including the planned Matjiesfontein station in Southern Africa. This network consists of both permanent and mobile (MOBLAS systems) stations ranging to SLR tracked missions. The ILRS isn’t responsible for the funding and operations of the stations however the host nation’s national research or government agency is in charge of funding operations and activities as well as mandating them.

The ILRS’s current global reference frame is designated the name SLRF 2008 (ILRS, 2013), which consists of the ILRS coordinates and velocity models of the ITRF 2008 reference frame. During ITRF definitions, these station positions are combined with other technique specific reference frames through local ties at co-location sites to estimate global and arc parameters for the satellite. This is the only technique for combining technique specific reference frames, via local ties, a technique which is analogous to space based co-location.

Each ILRS station has key components that include high energy pulsed laser, accurate timer, and a precise photon detector (Combrinck, 2010). Improvements to this network include dual colour ranging systems, addition of advanced stations in the network, automation and faster data delivery, improved refraction models and these are further explained in Gurtner et al. (2005). This is in order of increasing the yield in terms of data volume, better normal point precision as well as reducing operational costs.

Each station observes according to predicted orbits and satellite priorities then the data goes through a strict quality check procedure. All SLR data is submitted to the NASAs Crustal Dynamics and Data Information System (CDDIS). Before May 2012, the recognised data transfer format was the MERITX format which was then changed to the Consolidated Laser Range Data Format (CRD) which is currently adopted as the official format for SLR. After the data has been transferred to CDDIS, it will be made available to the AWG for processing, computation and analyses of the official ILRS products. The products obtained from the analyses are then transferred in SINEX format, the official format recognized by the ILRS and
IERS for storing estimated global parameters. These products will include site eccentricities which are essentially offsets between the monument location and the telescope’s reference point in both X, Y, Z and East, North, Up.

2.2 SLR OBSERVATION MODELLING

This subsection aims to introduce the reader to the various error sources in satellite laser ranging and their respective modelling. This process in satellite geodesy is termed observation modelling. The error sources are detailed below together with the complete range model. As a general rule in SLR, these error sources should be modelled with an accuracy that is one order of magnitude better than the accuracy with which the observations were obtained (Seeber, 2003). This means that if the accuracy of the SLR system is at the 1 mm level, then the corrections should at least be modelled with 0.1 mm accuracy.

2.2.1 MEASUREMENT ERROR SOURCES

The complete range equation for SLR can be written as, in metres (m):

\[ R_i = \frac{1}{2} \Delta t_i c + d\epsilon_i + dCoM_i + datm_i + db_i + d\epsilon_i \]

The equation above includes the standard corrections and biases applied to SLR data as recommended by the governing and overseeing organizations such as ILRS, IERS and IAG. On the left hand side \( R_i \) is the \( i^{th} \) measured range at a particular station. The measured time \( \Delta t_i \), should be measured to an accuracy of about \( 10^{-12} \) of a second (1 picosecond) in order to obtain high quality SLR results (Seeber, 2003). Currently, two timing systems are applied, the interval counters and event timers. Interval counters measure directly the time-of-flight of the laser pulse from station to reflector and back. On the other hand, event timers measure the pulse’s time-of-flight by differencing the epoch at which the pulse left the emitter and the epoch at which the reflected pulse arrives at the detector. Majority of the current systems currently utilize the interval counter system. The rest of the terms in this equation are discussed in the following subsections.
Site eccentricity ($d_{ei}$)

This is the offset between the SLR station and the ITRF monument on the ground. The site eccentricity must be known with accuracies at the 1 mm level or better (Seeber, 2003). The connection between the stations’ reference mark, which is essentially the intersection of the telescope exes and the monument, is made during high precision co-location surveys which are mandated by the IERS. These values are required to be updated especially in cases when the configuration of the instruments at the ranging site has changed. For permanent stations, this offset should be 0m due to the monument and the telescope’s reference being located at the same location.

Centre of Mass correction ($dCoM_i$)

There are two elements to understand with respect to this correction. The first is that there exists an eccentricity at the satellite due to the fact that the Corner Cube Reflectors (CCRs) are on the surface of the satellite and the Centre of Mass (CoM), which the satellite’s orbit refers to, is the satellite centre of gravity. For the geodetic spherical satellites, such as LAGEOS and ETALON, this value should, theoretically, remain constant and thus it is modelled with high accuracies for these satellites because their mass doesn’t change meaning their CoM won’t change as well, which then results in a constant CoM offset. However, for irregularly shaped satellites such as those of GNSS and remote sensing, this offset changes due to variations in propellant volume on board the space craft. For these satellites, the CoM offset values are updated by the respective Mission Control Centres (MCCs).

The modelling of CoM offset involves careful engineering measurements, an establishment of the relationship between that coordinate system which is arbitrarily chosen at the satellite (satellites’ reference frame), the satellites’ CoM, and the optical centre of the CCRs (ILRS, 2013). Figure 1.10 depicts this configuration. From the Figure, the orange and the green vector are obtained via engineering measurements pre-launch, and a suitable application of vector algebra to obtain the offset between the optical centre of the reflector and the satellites’ CoM.
The second element to this correction is the fact that, due to detector capabilities on the ground, there is a residual error introduced to the CoM offset and is added (or subtracted) to the standard CoM offset mentioned above. This means that this error is unique to each station in the ILRS network, and the values are made available to the analyses centres and are published by the ILRS and can be incorporated in the processing software. This error is dependent on station specific properties thus it is sometimes termed detector dependent. For example, the standard value for a LAGEOS satellite is about 251 mm however due to the laser ranging system used; there will be a departure of about 2 mm, as Combrinck (2010) points out. For an adequate modelling of this correction, information about the size and shape of the CCRs, satellites’ orientation, and the characteristics of the laser ranging system on the ground must be known (ILRS, 2013). The detector dependent corrections are decided upon during the deliberation of ILRS AWGs and are applied for certain stations. The detector dependent values applied for this study range from -0.006 mm to +2 mm for LAGEOS satellites and 0 mm to -6 mm for ETALON satellites as decided by the ILRS AWGs for realisation of the ITRF 2008.

**Atmospheric refraction (\(datm_i\))**

This effect remains one of the major forms of error sources for SLR as it uses optical wavelengths. The effect that the atmosphere has on optical wavelengths is twofold. Firstly, the atmosphere slows down the pulse as it travels to and from the satellite, and secondly it
bends the laser light, resulting in the measured range appearing longer than the geometrically (ideal) path. In correcting for this effect, the station coordinates are transformed to a topocentric coordinate system so as to consider the space between the telescope and the satellite, and then a standard model is applied. The standard model recommended for SLR is the Marini-Murray refraction model which was developed and published in 1973 by J.W. Marini and C. W. Murray, Jr. The original formulation of the model is as follows:

\[
datm_i = \frac{f(\lambda)}{f(\varphi, H)} \cdot \frac{A + B}{\sin E + \frac{B/(A + B)}{\sin E + 0.01}}
\]

Where:

\[A = 0.002357P_0 + 0.000141e_0\]

\[B = (1.084 \times 10^{-8})P_0T_0K + (4.734 \times 10^{-8}) \frac{P_0^2}{T_0} \frac{2}{(3 - \frac{1}{R})}\]

\[K = 1.163 - 0.00968 \cos 2\varphi - 0.00104T_0 + 0.00001435P_0\]

\(E\) – Satellites’ elevation angle in degrees

\(P_0\) – Local atmospheric pressure at the ranging site in mbars

\(T_0\) – Local temperature at the ranging site in Kelvin

\(e_0\) – Partial water vapor pressure at the ranging site in mbars

\(f(\lambda)\) - Laser frequency parameter and \(\lambda\) is the wavelength in \(\mu m\) (micro-metres) and is given by:

\[f(\lambda) = 0.965 + \frac{0.0164}{\lambda^2} + \frac{0.000228}{\lambda^4}\]

\(f(\varphi, H)\) - The ranging site function in terms of the stations’ latitude and the stations’ height and is given by:

\[f(\varphi, H) = 1 - 0.0026 \cos 2\varphi - 0.00031H\]
According to Seeber (2003), this model had uncertainties at the 1cm level in zenith delay (ZD) terms and the accuracy degrades at lower elevations. The Marini-Murray model is valid for elevation angles greater than ten degrees (Marini & Murray, 1973).

Due to the lack of separability of delays in the zenith direction (ZD) and Mapping Functions (MF), the Marini-Murray model required improvement of zenith delay terms and mapping functions. Although there were numerous studies and improved models proposed prior 2000, however the improvements by Mendes et al. (2002) and Mendes & Pavlis (2004) are preferred due to the yield of high quality results. This improvement was twofold: The first improvement was in the mapping functions (MF) by Mendes et al. (2002) and the second improvement was that of the zenith delay terms (ZD) proposed by Mendes & Pavlis (2004). Mapping functions (MF) describe the dependence of atmospheric refraction on the satellites’ elevation angle. The improved MFs cover a wide range of elevation angles with six degrees as minimum however the standard twelve degrees is set as a limit for SLR data processing.

The combination of the new improved zenith delay prediction models and mapping functions was adopted by the ILRS AWGs in 2006 as the new standard model to be applied in SLR data as of 01 January 2007 (Petit & Luzum, 2010). Another improvement in the modelling of refraction comes in the form of two-colour ranging which utilizes two laser pulses of different wavelengths.

**Signal delay correction \( (db_i) \)**

This term includes the range and time biases. The time bias refers to the delay in the registration of the returning pulse. This means that the epoch at which the pulse returned to the detector was recorded some time later. The time bias has to be deduced from the registered epoch of detection of the pulse. The correction for time biases is not applied to every single station in the ILRS network, but for certain stations after an assessment of the stations’ clock is carried out, a process that involves a comparison of each stations’ clocks’ frequency with the known frequency standard. Another way to correct for the time bias is calibrate the clocks utilised at the station.

The range bias is the length by which the range has been measured too long (or too short) and therefore should be subtracted from the estimated range. As mentioned in the introduction, the ILRS utilizes two bias measures, amongst others, to qualify SLR network stations. These are
the short term bias stability, which is the RMS of the pass-by-pass range biases, and the long term bias stability, which is essentially the standard deviation of monthly short term bias stabilities. There are various ways of treating range biases, one approach is to neglect range biases and not account for range biases at all, another approach is to estimate the range biases and apply it to a-priori data, or estimate weekly range biases (Coulot et al., 2006). The longer or shorter the ranges are measured, the higher, or lower the station height appears to be. Similar to time biases, range biases are estimated for some stations only in the ILRS network due to the correlation between range biases and the stations height. In literature, it is mentioned that range biases emanate from multiple sources including errors in atmospheric gauges at the ranging site (used to model refraction), unstable calibration targets, signal strength issues, detector properties etc. (Combrinck, 2010) as well as the degradation of the clocks’ frequency.

Unmodeled residual and random effects \((d\varepsilon_i)\)

The errors included in this category are both systematic and random in nature. The systematic part includes errors emanating from mechanical instabilities, misalignment etc. of the laser ranging system such as pointing errors as well as other mechanical tendencies. A majority of these errors are calibrated internally at the ranging site and the calibration information published to ILRS.

Other corrections

The modelling of SLR ranges usually involves general relativistic modelling which is due to the rotation and motion of the Earth and the motion of the satellite in space. The modelling of this effect is not covered in this study but reference is made to Petit & Luzum (2010) for relevant models for recommended by the IERS.
3. ORBIT MODELLING

3.1 THE ORBIT DETERMINATION PROBLEM (TWO BODY PROBLEM)

In celestial mechanics, the two body problem is used to model the motion of two objects under mutual gravitational attraction in space. When dealing with artificial satellites, the mass of the satellites is neglected in the formulation of the resulting equations of motion. In this particular case and for the purposes of this study, only the Earth and artificial satellites are considered, however there are higher orders of the two body problem *i.e.* three body problem and N-body problem in which the interactions between 3 or more objects are taken into account when modelling their relative motion. The formulation of the two body problem is detailed in the following paragraphs with accompanying equations. It should however be stated that the fundamental idea in orbit modelling is that, given the position and velocity of a satellite in the form of an Initial State Vector at an initial epoch \((t_0)\), determine the position and velocity of the satellite at any other time \((t_f)\) in its trajectory (Seeber, 2003), with an additional condition that the satellite must be observed, whether in the form of a range or Doppler shifts or carrier phase *etc.*

The initial epoch mentioned above is chosen to be precisely midnight Sunday in accordance within the GPS week in the case of SLR data collection and subsequent processing. The initial state vector is determined from the global range observations according to the range model dealt with in chapter 2. The derivation of the two body problem is done in a celestial reference frame. The assumption is that the bodies are homogenous and generate gravity of a point mass (Seeber, 2003), thus the particle model is used, meaning that the two object are considered as simple points in space. Since the initial conditions are unperturbed, the resulting orbit is also termed Keplerian orbit.

The derivation of the satellites’ equations of motion utilises two Newtonian equations, namely: Newton’s second law of motion stated as \(\ddot{\mathbf{r}} = m\dot{\mathbf{r}}\), and the universal law of gravitation stated as \(\ddot{\mathbf{r}}_{\text{net}} = -G \frac{Mm}{r^2}\). The two equations are equated and solved in component form for accelerations in the X, Y, and Z directions along the spatial distance between the two bodies, the Earth and the satellite in a celestial reference frame. The origin of the celestial reference frame is chosen to be the Earth’s centre of mass (geocentre). The full derivation of the two-body problem can be found in Seeber (2003). After relevant manipulations and
neglecting the satellites’ mass \((m)\), an expression of the satellites’ unperturbed motion in space with respect to the larger mass \(M_e\) (Earth) in the form of a second order differential equation:

\[
\ddot{r} = - \frac{GM_e}{r^2} \frac{r}{r}
\]

Where:

- \(r\) - The time dependent geocentric position vector of the satellite referring to the satellites’ centre of mass and \(\frac{r}{r}\) is the unit vector along the spatial distance between the satellite and the Earth.
- \(G\) - The universal gravitational constant \((6.7 \times 10^{-15} m^3 kg^{-1} s^{-2})\)
- \(M_e\) - The mass of the Earth, and
- \(\ddot{r}\) - The vector of acceleration of the satellite along the spatial distance between the two bodies.

The universal constant \(GM_e\) is also known as the absolute Earth scale or geocentric gravitational constant (Petit & Luzum, 2010) and is more precisely determined from SLR (Dunn et al., 2002) and is published together with defining parameters for geodetic Earth models such as WGS84 utilised in South Africa. The constant \(GM_e\) is a dynamic equivalent of the geometric scale factor determined during the ITRF definitions. Performing a dimensional analysis of the above equation, one would obtain \(m.s^{-2}\) as it is the time dependant acceleration of the satellite in space. The direction of the resulting acceleration is encoded in the negative sign and it is towards the origin of the celestial frame which is chosen to be the geocentre. A double integration of this expression will result in six constants of integration. These constants are the initial conditions representing the initial state vector in Cartesian coordinates and are written as \(X_0, Y_0, Z_0, \dot{X}_0, \dot{Y}_0, \dot{Z}_0\) (position and velocity at \(t_0\)) or in Keplerian form.

The orbit described thus far is an ideal orbit. In reality, the satellites’ orbit departs from the ideal orbit due to factors emanating from both gravitational and non-gravitational influences.
The resulting orbit is said to be perturbed or disturbed. This form of motion is detailed in the following section.

3.2 Purturbed Satellite Motion

The equation above is valid for a spherically symmetric Earth and thus gives an ideal orbit. However, due to the Earth’s rotation and unequal gravitational attractions due to the undulations of the surface as well other forces acting on the satellite, this is not the case. This results in the satellites’ orbit varying throughout its trajectory such that a new set of orbital elements has to be determined at chosen time steps. These are referred to as osculating ellipses and can be viewed as being enveloped by the entire satellites’ trajectory (Muller & Torge, 2012). The additional accelerations experienced by the satellite are included in the vector \( \ddot{r}_p \), denoting perturbing total accelerations and thus the new equation becomes, as presented by Govind (1994):

\[
\ddot{r} = -\frac{GM_e}{r^2} \frac{r}{r} + \ddot{r}_p
\]

Where:

\[
\ddot{r}_p = \ddot{r}_E + \ddot{r}_{3Body} + \ddot{r}_{SRP} + \ddot{r}_{Drag} + \ddot{r}_A + \ddot{r}_{Tides} + \ddot{r}_{\text{Gen Accel}}
\]

These perturbing accelerations are:

\( \ddot{r}_E \) – Acceleration due to Earth’s perturbing potential

\( \ddot{r}_{3Body} \) – Acceleration due to third body perturbations caused by Sun and Moon and other planets

\( \ddot{r}_{SRP} \) – Acceleration due to solar radiation pressure from the Sun

\( \ddot{r}_{Drag} \) – Acceleration due to atmospheric drag (for low altitude satellites)

\( \ddot{r}_A \) – Acceleration due to Earth’s albedo caused by reflective surfaces such as glaciers, ice caps etc.

\( \ddot{r}_{Tides} \) – Accelerations due to Earth tides and ocean tides

\( \ddot{r}_{\text{Gen Accel}} \) – General acceleration of the satellite.
During the orbit modelling procedure, the first step is to determine the initial conditions of the satellite at an initial epoch in a process referred to as orbit determination. These initial conditions are then integrated throughout the satellites’ trajectory for the perturbing accelerations at chosen time steps. Careful attention should be given to the size of the integration time steps in order to optimise accuracy and efficiency of the resulting accelerations. Too small or too large a step size will result in poor accuracy. The numerical integrators used are the Runge-Kutta or Euler methods of integration. In practice, numerical methods of integration are preferred compared to analytical methods due to ease of computer implementation (Montenbruck & Gill, 2000). Analytical methods have difficulties associated with them and these methods are utilized in the planning of satellite missions and pre-launch analyses (Torge & Muller, 2012). This integration stage is referred to as orbit generation and will produce a set of tabulated satellite ephemerides which are then transferred to the relevant users for subsequent processing and obtain desired products.

The interest of this study is towards that perturbation which emanates from the Earth’s geopotential ($\ddot{r}_E$) as this effect comprises the parameters defining the Earth’s geocentre. The modelling of this effect is covered in the following section.

### 3.3 SLR DYNAMIC GEOCENTRE DETERMINATION

The greatest perturbation of a satellites’ orbit is that due to the Earth’s non-symmetrical gravitational field. Within the Earths’ total gravitational potential, the Earths’ oblateness has a larger and noticeable effect on satellite orbits. This effect is expressed by the zonal harmonics $J_n$ with $J_2 (-C_2)$ termed the dynamic form factor and an important parameter in defining global Earth models such as the WGS84. For low Earth satellites at altitudes of 300 km to 20000 km (Wellenhof & Moritz, 2005), this effect has the tendency of inducing a torque on the satellites’ orbital nodes towards the equator (Seeber, 2003). The paragraphs that follow detail the equations that are used and later, a method for extracting geocentre coordinates will be detailed. The total geopotential of the Earth is given by the expression:

$$U = \frac{GM_e}{r} + R$$
The first term represents the geopotential of a spherically symmetric Earth and will result in an orbit such as that described by Kepler’s three laws and is termed the Keplerian term (Seeber, 2003). The second part is the perturbing potential which is analogous to, but should not be confused with the disturbing potential $T$, used in the terrestrial geopotential models. The acceleration induced by geopotential upon the satellite is then given by $\ddot{r}_E = \nabla U$ or simply put; gravitational acceleration is the gradient of gravitational potential. Since the perturbing potential is harmonic, it is represented by a spherical harmonic expansion in a terrestrial reference frame resulting in the equation:

$$U = \frac{GM_e}{r} \left[ 1 + \sum_{n=1}^{n_{\text{max}}} \left( \frac{R_e}{r} \right)^n \sum_{m=0}^{n} \left( \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda \right) P_{nm}(\cos \theta) \right]$$

Where:

$(r, \theta, \lambda)$ – Geocentric spherical coordinates of the point of interest

$R_e$ – Mean radius of the Earth ellipsoid

$P_{nm}(\cos \theta)$ – Legendre’s functions of the first kind

$\bar{C}_{nm}, \bar{S}_{nm}$ – Fully normalised Harmonic coefficients of the Earth’s geopotential of degree $(n)$ and order $(m)$. These coefficients describe the mass distribution within a central body, the Earth in this case. Note that the double summation starts from $n = 1$ as the degree one coefficients are treated as unknowns in this study, however, for terrestrial geopotential models, the coefficients in $n = 0, 1$ are treated as zero as these define the origin. The coefficients are zonal for $m = 0$, tesseral for $m \neq 0$, and sectorial for $m = n$

For this study, the highest degree $n_{\text{max}}$, is 200 and $m_{\text{max}}$ is 160 (GGM02C gravity field model). However, due to the specific interest in the lower degree terms, the maximum degree and order estimated for in this study is 10. Theoretically, the double summation is carried out up to infinity to cover the entire surface of the Earth. However, due to the negligible influence of higher degree and order coefficients, the summation is performed up to a geopotential
specific degree (n) and order (m) limit. The higher this limit, the better the resolution of the geopotential model. Geopotential coefficients are normalised to avoid computational instability. However, in order to add meaning to the values of these coefficients, they have to be un-normalised. By definition, the centre of the spherical coordinate system is transferred to the geocentre and the Z axis coincides with the rotational axis of the Earth and the coefficients in n = 1 and m = 0, 1 are equal to 0.

With respect to ITRF definitions, the geocentre is defined by determining the translational parameters \((T_x, T_y, T_z)\) at epoch during the multi-technique combinations. This is the geometric determination of the geocentre. For a meaningful analysis and interpretation for the purpose of this study and similar studies, the degree 1 geopotential coefficients are used to obtain the geocentre dynamically. In turn, a physical interpretation about the behaviour of the geocentre can be derived. These coefficients are then un-normalised by multiplying by the constant \(\sqrt{3}\), which only applies to the degree 1 coefficients only, and a different un-normalising constant applies for higher degree coefficients. The a-posteriori values are then converted to linear units via the formulae, where \(R_e\) represents the radius of the Earth and \(X_{com}, Y_{com}, \) and \(Z_{com}\) are the Cartesian coordinates of the geocentre:

\[
X_{com} = \sqrt{3}R_e \hat{C}_{11} \\
Y_{com} = \sqrt{3}R_e \hat{S}_{11} \\
Z_{com} = \sqrt{3}R_e \hat{C}_{10}
\]

These are the so called scaled coordinates of the geocentre (Govind, 2010) in mm. Converting these coefficients in cartesian form provides a more intuitive feeling of the results and allows better visualisation during analysis. As stated previously, these coefficients are theoretically zero. This means that their values are registered as zero in the global terrestrial geopotential models such as the GGM02C gravity model which will be utilised in this study. For the satellite solutions, the zero values are used as a-priori values which will populate the design matrix together with higher degree and order coefficients. After adjustment, the a-posteriori values will be corrections to the zero a-priori values and can be written in the form
(Δ\tilde{C}_{11}, Δ\tilde{S}_{11}, Δ\tilde{C}_{10}). These corrections to the null geocentre are indicative of geocentre motion in X, Y and Z directions for the epoch of the estimation.

Factors causing geocentre motion are both periodic and seasonal in nature. These geocentre variations are usually in the order of a few millimetres with the Z components reaching as much as 20 mm. These include mass redistribution such as glacial ice melting, sea level rise, and ground water redistribution within the Earth’s continents etc., configuration and distribution of SLR network stations. It is clear that by studying the dynamically determined geocentre, studies about natural processes such as global warming and climate change can be derived from the analysis of the seasonal and periodic variations of the geocentre by performing a long term spectral analysis due to the long term sensitivity rather than short term sensitivity to climatic processes. An important technical detail to note is that, the spherical harmonic expansion is in a terrestrial reference frame and the orbit modelling process is performed in a celestial reference frame, this implies that the harmonic expansion is transformed to a celestial reference frame for uniformity during orbit modelling.
4. METHODOLOGY AND DATA DESCRIPTION

This section details the data used and the method that is followed in the investigation and analysis thereafter. Initially, the data components that are utilised during the investigation are described prior to detailing the analysis procedure. It should be kept in mind that the data set detailed in this section and the computational procedures and software both comply with standards published and agreed both by the ILRS and those set out in the IERS conventions (Petit & Luzum, 2010). The study utilizes data from the 4 geodetic satellites: LAGEOS-1, LAGEOS-2, ETALON-1 and ETALON-2.

4.1 BASELINE DATA AND SOLUTION

This component of the study is composed of a 1 year time series of SLR orbit solutions from the beginning of 2011 to the end of 25 December 2011 computed from observations obtained by the ILRS global network of SLR stations. These orbit determination solutions were computed in weekly arcs for the same period. These solutions are equivalent to the official solutions submitted to the ILRS for its official combined weekly product within the period of this study with the exception that, the weekly geocentre solutions are determined dynamically/ gravimetrically where as those submitted to the ILRS are computed geometrically by estimating the translational parameters during a seven parameter transformation. It was thus possible to utilise the ILRS geometric geocentre solutions in order to validate the results obtained within this study. The convenience in doing so was the inclusion of an independent check on the results of this study.

4.2 SIMULATED DATA

The initial stage prior the simulation process was to obtain 3 dimensional GPS coordinates for the location which is proposed to be in Matjiesfontein referred to as Matj9999 in the processing steps hereafter. These coordinates, supplied by Combrinck (2014), are as follows:

- \( X = 4998485.7294 \) m,
- \( Y = 1876789.8194 \) m,
- \( Z = -3479352.8976 \) m.
This GPS position was applied in conjunction with the orbit solutions determined from the baseline data to derive perfect range observations for the Matjesfontein station (Matj9999). During the simulation, the important elements to consider were that the simulated data should not dominate the SLR orbit solutions, and that they should be as realistic as possible.

The solution to this was to generate the data such that in terms of data volume, the data obtained for Matj9999 is within the data obtained by the two of the best performing SLR stations in the ILRS global network namely: Zimmerswald in Switzerland and Yarragadee in Australia. Factors such as the number of cloud free days, stations’ operational times and days are taken into account in order to generate realistic data. The second solution was to add a reasonable noise level to the simulated range observations. This noise level was decided after an experimentation process where, after each simulation, the RMS (Root Mean Square) of the simulated observations had to resemble that of the best performing stations in the global ILRS network, after the orbit estimation procedure using this simulated data. Note that the resemblance in case doesn’t imply that this RMS should be exactly the same but it is allowed to depart by as much as 2mm from that of the best performing stations. In layman’s terms, this means that the simulated observations should have a resembling consistency to the best performing stations for the reasonability of the orbit solutions determined thereof. The next step was to limit the simulated observations by limiting the daily simulation times, removing data generated on Sundays and public holidays ensuring that the simulated data does not dominate the orbit and the geocentre solutions. The data limits for each satellite are summarised in Table 2 below.

<table>
<thead>
<tr>
<th></th>
<th>LAGEOS 1 &amp; 2</th>
<th>ETALON 1 &amp; 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Noise level</strong></td>
<td>0.05 – 0.1 cm</td>
<td>0.1 – 1 cm</td>
</tr>
<tr>
<td><strong>Observation Interval</strong></td>
<td>120s (2min)</td>
<td>300s (5min)</td>
</tr>
<tr>
<td><strong>Elevation cut-off angle</strong></td>
<td>15 deg</td>
<td>20 deg</td>
</tr>
<tr>
<td><strong>Operation</strong></td>
<td>6 days, 9Hrs</td>
<td>6 days, 9Hrs</td>
</tr>
</tbody>
</table>

The simulated dataset is generated for single wavelength light as opposed to dual colour ranging, and the wavelength was set at 0.5320 μm. The Observation interval is the intervals at which the normal points to each satellite are generated. In GEODYN II (Pavlis et al, 2006), the contributors to the noise level are not separable when simulating data, this is the reason...
the strategy outlined above was chosen when selecting the reasonable noise levels for each satellite.

4.3 ANALYSIS PROCEDURE

The first stage is the orbit determination stage for the global network for the period selected for this study. The weekly orbit estimations were carried out using GEODYN II (Pavlis et al., 2006) software and the satellite solution combinations was carried out in NASA SOLVE (Ullman, 1994)to estimate weekly geocentre estimates with all 4 satellites combined. The weekly harmonic coefficients of the Earth’s geopotential obtained from this procedure were then converted to Cartesian coordinates by utilising the equations shown in section 3.3. The processing procedure utilised in this study was three fold:

- Process the baseline data to obtain the baseline solution (already includes HartRAO MOBLAS-6’s data). The baseline solution is described in section 4.1 above.
- Combine the baseline data and the simulated data of Matj9999 station and process treating Matj9999 as a core site.
- Process the baseline data combined with data from Matj9999 and remove HartRAO MOBLAS-6 from this set of solutions such that the solutions obtained only involve Matj9999 and the global network.

In the processing steps above, the results were analysed for geocentre variations due to these three configurations and compared to verify the improvement in the level of uncertainties.

4.4 PROCESSING WORKFLOW

The first stages of this study involved orbit estimation in weekly arcs to determine satellite specific parameter which form part of the perturbing accelerations that are induced by various forces both gravitational and non-gravitational which were mentioned in section 3.2. This set of parameters is termed the ARC set parameters. The ARC set parameters were estimated together with the GLOBAL set parameters which are specific to the frame of reference, the SLRF 2008, which is that subset of the ITRF 2008 global station coordinates composed only of ILRS stations. These two sets of parameters were estimated in an iterative process in weekly arcs until convergence was reached in a least squares estimation process utilising
GEODYN II (Pavlis et al., 2006) software package. The main output of this process are the weekly partials of the orbit estimation parameters also referred to as the design matrices and weekly generated orbits, and the setup files from each satellite after the orbit estimation procedure. The importance of the satellite setup files is towards the simulations for Matj9999 station as they contain the computational models which were utilised in the orbit estimation of the baseline data. In essence these setup files are orbit solutions from which the generated orbits are obtained. The weekly satellite specific partials were then inputs in the next important package of the study, the NASA SOLVE (Ullman, 1992) program which is a companion to GEODYN, where the satellite specific partials are combined for weekly combined geocentre solution. The diagram below summarises this multistep processing procedure.
Figure 12: A diagrammatic representation of the processing scheme adopted in this study with the simulation data flow depicted alongside as well.

For convenience, this diagram includes the simulation stage by way of illustrating where this process takes place during the entire processing workflow. However it should be noted that the simulation procedure only occurs once, it is not an iterative process. As can be seen from the process flow diagram, the simulated observations are then combined with the baseline data and then the orbit estimation procedure restarts, then those results are taken forward to the multi-satellite combination stage.
4.5 ILRSA SOLUTION

This solution is a 12 month solution published in weekly arcs covering the period from 01 January 2011 to 24 December 2011. This is the solution that will be utilised to validate those solutions obtained within study. This set of solutions essentially contains weekly translational parameters obtained via a seven parameter transformation which is then combined with the rest of the ILRS solutions to make the official combined weekly product delivered to the users. The solutions to the seven parameter transformation are obtained via constrained solution of the ILRS core network. Figure 4.2 below depicts the ILRSA geometric solution for the 1 year 7 day arc geocentre solutions.

![Weekly ILRSA Translational Parameters in X, Y, Z Directions](image)

**Figure 13:** ILRSA geometric solutions depicted in X, Y and Z components.

The comparison of these solutions with those obtained in this study is detailed in chapter 5. The ILRSA in the analyses will be termed ILRSA_SOL and the solutions of this study will be termed DYNAMIC_SOL for the sake of differentiation.
4.6 THE COMPUTATION MODELS

Up until this point, it has been mentioned that numerous forces have a noticeable influence on the satellites in space, the observing SLR stations, and the medium between the satellite and the observing station. The computational models that will be stated in this section describe these forces, utilising empirical data collected on various campaigns with the intent of describing the nature of these forces. These models are reviewed and revised periodically by the overseeing organisations considering the level of accuracies required by the IERS in its products. In this study, the models utilised comply with the IERS Conventions 2010 (Petit & Luzum, 2010). There are two sets of computational models that have been utilized in this study, namely the computation models applied in orbit modelling using the original global network and that computation model applied in generating the simulated observations for Matj9999 station.

A decision was made to not include atmospheric gravity in the processing of the baseline data and subsequent solutions. The tropospheric refraction model applied to the observations is the Marini-Murray model (Marini & Murray, 1973) together with Mendes et al. (2002) mapping functions and zenith delays by Mendes & Pavlis (2004), in keeping with the guidelines of the IERS conventions (Petit & Luzum, 2010). The applied Centre of Mass (CoM) correction to the satellite observations is +0.251m for both LAGEOS satellite and +0.576m for both ETALON satellites, note that these values represent the radii of the satellites as they are spherical by design. The detector dependent departures, as per ILRS AWG standards, were then applied to these CoM offset for each station in the tracking compliment. The editing RMS is determined by the RMS of the weighted weekly observations for each satellite. Each editing value from the weekly arcs is then, multiplied an editing multiplier of 3.5 which is standard for SLR. The minimum elevation angle was set at 12 degrees.
Table 3: The computation models used in the processing of this study categorised as Orbit models, Station position and Reference frame, and estimated parameters.

<table>
<thead>
<tr>
<th>1. ORBIT MODELS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth’s Gravitational Potential</td>
<td>GGM02C - degree and order 160 x 160</td>
</tr>
<tr>
<td>Solid Earth Tide Potential</td>
<td>IERS Conventions 2003</td>
</tr>
<tr>
<td>Ocean Tide Potential</td>
<td>Ray GOT4.7</td>
</tr>
<tr>
<td>Third Body Perturbations</td>
<td>Sun, moon and planets physical values (GM, 1/f, a),</td>
</tr>
<tr>
<td></td>
<td>JPL DE410 planetary ephemeris</td>
</tr>
<tr>
<td>Direct Solar Radiation Pressure</td>
<td>Cannonball</td>
</tr>
<tr>
<td>Integrator</td>
<td>Cowell 11th order</td>
</tr>
<tr>
<td>Atmospheric Gravity</td>
<td>Not Applied</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. STATION POSITION AND REFERENCE FRAME</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nutation and Precession</td>
<td>IERS 2003</td>
</tr>
<tr>
<td>EOP</td>
<td>IERS C04/08 apriori values daily</td>
</tr>
<tr>
<td>Plate Motion</td>
<td>none</td>
</tr>
<tr>
<td>Planetary and Lunar Ephemeris</td>
<td>JPL DE410</td>
</tr>
<tr>
<td>Station Displacement - Solid Earth Tide Load</td>
<td>IERS Conventions 2003</td>
</tr>
<tr>
<td>Station Displacement - Ocean Tide Load</td>
<td>GOT4.7</td>
</tr>
<tr>
<td>Reference Frame</td>
<td>ITRF2008 (SLRF08)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. ESTIMATED PARAMETERS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Global</td>
<td>Station Positions</td>
</tr>
<tr>
<td></td>
<td>Geopotential coefficients to degree and order 10.</td>
</tr>
<tr>
<td></td>
<td>Daily LoD and pole position</td>
</tr>
<tr>
<td>3.2 Arc</td>
<td>State Vector for all Satellites</td>
</tr>
<tr>
<td></td>
<td>1 Solar Radiation Pressure scale factor per week</td>
</tr>
<tr>
<td></td>
<td>Satellite Accelerations</td>
</tr>
<tr>
<td></td>
<td>Range bias – Estimated as per AWG rules</td>
</tr>
<tr>
<td></td>
<td>Time bias – none estimated</td>
</tr>
</tbody>
</table>
5. RESULTS AND ANALYSIS

This chapter presents the results and analysis of the processing strategies outlined in the previous section. This set of results comprises the three combination strategies introduced in the previous chapter. Each set of results is presented separately in terms of the X, Y and Z geocentre components together with weekly uncertainties. Usually, the level of uncertainty in an estimate is understood to be the range within which this estimate is expected to lie. Uncertainties should represent all possible sources of errors. Thus if the level of uncertainty is miniscule, then the influence of these errors both systematic and random is also miniscule resulting in a better estimate. This is thus the justification for utilising the uncertainties as the key quality indicator during the processing in this study. For validation, these results are compared to the weekly series of geometric geocentre estimates obtained from the ILRS termed ILRSA_SOL.

It is common practice in SLR to obtain the solutions via minimum constraint estimations, however for this study, the station coordinates of the core network have been constrained, meaning that only non-core station coordinates are estimated during all three processing strategies. Throughout this study, the station coordinates of the core network have been fixed to their SLRF 2008 frame values at epoch of observation. The component of this study that comprises the results and analysis of the orbit estimation procedure will not be presented. As mentioned earlier, this is essentially a study of geocentre motion; however no spectral analysis is possible with the amount of data utilised, which is a one year series of weekly arcs. This implies that no seasonal, periodic or secular trends in geocentre motion can be identified due to long term sensitivity of these trends; however, the analyses are based on short term trends emanating from a change in the network configuration, that is, shifts in the dynamic geocentre estimates.

5.1 SOLUTION #1: BASELINE SOLUTION

This section presents 12 months of weekly 7 day arc solutions of the dynamic geocentre estimates from the 4 geodetic satellites, ETALON 1 & 2 and LAGEOS 1 & 2. Figure 14 shows the weekly variation in the geocentre results together with the weekly uncertainties of the CoM coordinates. The weekly uncertainties are shown in magenta in Figure 14. As termed, these are the baseline solutions from which the subsequent solutions will be compared.
and the level of improvement drawn, due to the fact that they represent the original conditions as defined by the original observations. Table 4 depicts the quality indicators between the ILRSA_SOL (Figure 13) and DYNAMIC_SOL drawing the mean and RMS geocentre estimates from these two solutions.

Table 4: Mean and RMS of centre of mass coordinates determined from DYNAMIC_SOL and ILRSA_SOL solutions.

<table>
<thead>
<tr>
<th>SOLUTION</th>
<th>Xcom (mm)</th>
<th>Ycom (mm)</th>
<th>Zcom (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DYNAMIC_SOL</td>
<td>MEAN (-0.1)</td>
<td>MEAN (-4.3)</td>
<td>MEAN (-0.6)</td>
</tr>
<tr>
<td></td>
<td>RMS (3.6)</td>
<td>RMS (5.1)</td>
<td>RMS (9.2)</td>
</tr>
<tr>
<td>ILRSA_SOL</td>
<td>MEAN (2.6)</td>
<td>MEAN (1.9)</td>
<td>MEAN (0.1)</td>
</tr>
<tr>
<td></td>
<td>RMS (5.3)</td>
<td>RMS (3.8)</td>
<td>RMS (7.1)</td>
</tr>
</tbody>
</table>

Table 4 above represents an initial step in the validation process of the solutions from this study, the DYNAMIC_SOL in X, Y and Z components. The means of these two independent solutions agree to within 2.5 mm in X and Y components and 0.5mm in Z. The RMS values of these solutions agree to within 2 mm in all components, meaning that these two solutions are consistent with each other. To derive more detail of variation between these two sets of solutions (DYNAMI_SOL vs ILRSA_SOL), the weekly differences in geocentre estimates were estimated. The mean and RMS of differences in geocentre estimates were utilised as the quality indicators of the calculated differences. The mean of geocentre differences was observed to be 2.6, 6.3 and 0.7 mm in the X, Y and Z directions respectively. This means that on average, there is a 2.6 mm agreement between the geometric and the dynamic X components and an average agreement of 6.3 and 0.7 mm in the Y and Z directions respectively. The RMS values of these differences are 6.9, 7.9 and 12.7 mm in X, Y and Z components respectively, denoting the consistencies of the offsets between the two solutions. Paying more attention to the dispersion of the two sets of solutions, Table 5 below depicts the ranges of geocentre estimates.
Table 5: Ranges derived from the ILRSA_SOL and DYNAMIC_SOL

<table>
<thead>
<tr>
<th>SOLUTION</th>
<th>Xcom (mm)</th>
<th>Ycom (mm)</th>
<th>Zcom (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DYNAMIC_SOL</td>
<td>18.8</td>
<td>11.2</td>
<td>59.3</td>
</tr>
<tr>
<td>ILRSA_SOL</td>
<td>18.8</td>
<td>15.5</td>
<td>35.4</td>
</tr>
</tbody>
</table>

Table 5 above represents the statistical range for geocentre estimates calculated as the difference between the smallest and the largest values in the both series of estimates. These values should give the reader an idea of the dispersion of the estimates in both solutions. It can be seen that the ILRSA_SOL series is 4.3 mm wider than the DYNAMIC_SOL for the Y estimates, whereas the DYNAMIC_SOL Z series is 24.0 mm wider in the Z component. This in fact shows the inherently large variation with respect to the motion of the Z component in geocentre estimates rather than noting the quality with which the Z component is determined. This means that compared to the X and Y component, this range implies the sensitivity of the Z component with respect to the equatorial plane. This can also be seen in the mean and RMS estimates of the dynamic solution in Table 4 above.

This inherent motion in the Z component is depicted in Figure 14 below together with the X and Y components of the DYNAMIC_SOL and their weekly uncertainties in all three components. It should be noted that Figure 5 depicts the physical motion of the geocentre as it is directly determined from the degree 1 harmonic coefficients of the gravity field.
Figure 14: DYNAMIC_SOL for the period 2011 to the end of April 2012.

As mentioned before, the uncertainties in the dynamic solutions represent the quality with which the dynamic solutions have been determined, which intern imply the quality with which these estimates define the dynamic geocentre. As expected, the majority of uncertainties are close to 0 mm. This is further supported by the magnitudes of the mean and RMS of the uncertainties. Both the X and Y components are estimated with uncertainties at the sub-millimetre level with 0.7 and 0.6 mm respectively, whereas the Z components have uncertainties at the 1.9 mm level, implying sub-millimetre accuracy in the determination of the X and Y components, which directly translate to the obtainable accuracies with the data utilised in these solutions. Further, the RMS values for the weekly uncertainties are at the sub-millimetre level for both the X and Y components, with 0.7 and 0.7 mm respectively and the Z component with the RMS of uncertainties at 1.9 mm. These RMS values can be understood to depict the spread in the distribution of the uncertainty measures.

By looking at the mean and RMS of uncertainties it is easy to see that the results obtained within this solution are up to standard and in accordance with the expected accuracies published by the IERS. However, there were a few arcs that yielded poor results in the DYNAMIC_SOL. These can be observed from figure 14 above to have large uncertainties and characteristic spikes in the estimated geocentre coordinates. Following a careful analysis
of the total observations contributed by these arcs, it was discovered that although the RMS of the overall observations were below 1 cm, these arcs resulted in poor accuracies. The reason being was the limited observations available for all satellites in these arcs. The spike seen in the uncertainties of the Y component emanates from the arc of 17 July 2011. This implies that there is a relationship between the total observations contributed by the global network in any given week. The more observations that proceed to the final SOLVE solution, the lower the uncertainties of the geocentre estimates. The next section presents the solutions in which the simulated data from Matj9999 has been included in the estimation of the geocentre estimates.

5.2 SOLUTION #2: BASELINE DATA + SIMULATED DATA

This set of results presents that set of results which contains the simulated data to produce a new one year time series of geocentre estimates. These solutions are presented for both LAGEOS and ETALON satellites and were not validated with respect to the ILRSA_SOL, but compared to the baseline solution dubbed DYNAMIC_SOL. From here onwards, the solutions contained in this section will be termed SIM_SOL. Matj9999 was treated as a core site meaning that the data from this simulation had more influence on the final results as well as other core sites. The realisation of an SLR station as a core site is in the weighting of its observations. In this study SLR core sites are weighted with sigma values 1.0 m and non-core sites with 10.0 m.

During the simulation, the aim was to generate the data such that Matj9999 generates equivalent observations as the best performing stations in the network. Another factor to mind was that this data should be as realistic as possible. The data was generated in weekly arcs and merged in GEODYN II (Pavlis et al., 2006) to produce combined observations which proceeded to the orbit estimation process and subsequent SOLVE solutions. The results from this estimation are presented graphically in figure 15 below. The mean and RMS values for those geocentre estimates obtained from this data set are presented in Table 6 below.
Table 6: Mean and RMS of SIM_SOL geocentre estimates

<table>
<thead>
<tr>
<th>SOLUTION</th>
<th>Xcom (mm)</th>
<th>Ycom (mm)</th>
<th>Zcom (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIM_SOL</td>
<td>MEAN (-0.6) RMS (3.2)</td>
<td>MEAN (-3.4) RMS (4.3)</td>
<td>MEAN (-1.1) RMS (8.5)</td>
</tr>
</tbody>
</table>

It is evident from the Table 6 above that there is a sub millimetre agreement between the X components of the DYNAMIC_SOL and SIM_SOL with a 0.5 mm agreement. The Y and Z components agree to within 0.9 and 0.5 mm respectively. These values validate the SIM_SOL against the DYNAMIC_SOL to ensure that there wasn’t any degradation of the geocentre estimates in the SIM_SOL solution as a result of introducing new data. The RMS values show a decrease of 0.4, 0.8 and 0.7 mm in X, Y and Z values compared to the RMS values from the DYNAMIC_SOL as shown in Table 4 which show more consistency in the SIM_SOL. This improvement in the distribution of the geocentre estimates is further shown by the statistical ranges of geocentre estimates with 16.8, 11.1 and 48.5 mm in X, Y and Z components respectively. This shows a decrease of 2 mm, 0.1 mm and 10 mm in X, Y and Z components. However, the change in the geocentre estimates themselves does not indicate any form of accuracy in the SIM_SOL, but is indicative of the dependence of the estimates with respect to the network configuration.

![Weekly Dynamic Geocentre (CoM) Estimates in X, Y, Z Directions & Uncertainties from 02 Jan 2011 to 25 Dec 2011](image)

**Figure 15:** SIM_SOL geocentre estimates for 2011
A more important statistic is the uncertainties of the estimates in the SIM_SOL solutions as this will show the improvement in geocentre estimates from the DYNAMIC_SOL solution. These values are depicted above in Figure 15. The mean of the weekly uncertainties was observed to be 0.6, 0.6 and 1.8 mm in the X, Y and Z directions respectively. The RMS values for uncertainties were observed to be 0.6 mm for X and Y and 1.8 mm in the Z direction. These solutions show a minimal improvement in the determination of the dynamic geocentre estimates. It is evident from the mean of uncertainties, the average improvements in these estimates is 0.1 mm in X, Y and Z directions. However, for considerably large data sets, this improvement would be considerably higher than estimated in this set of solutions.

There were however challenges in making sure that the data volume for Matjiesfontein in weekly solution remained at the same level as Zimmerswald (Switzerland) and Yarragadee (Australia). Amongst these, the key issue was to avoid loss of data due to editing in the orbit estimation process. If the observations for Matj9999 had good quality (RMS < 2cm), most observations from other stations would be removed during editing due to Matj9999 overwhelming the other stations. On the other hand, if the editing RMS was below Matj9999 RMS, most observations from Matj9999 would be removed. Due to this, an editing RMS of 1 – 2.5 cm was acceptable to allow as much data as possible for Matj9999. As can be seen from Table 2, the two elements used to control the data limits for Matj9999 were the elevation angle, and implementing the operation times by deleting data at set intervals in GEODYN II (Pavlis et al, 2006). In some arcs, there was considerable loss of data from Matj9999 due to editing, and in other arcs, the balance was achieved by setting a suitable noise level such that Matj9999 maintains similar quality as Zimmerswald and Yarragadee with respect to the RMS of observations.

To conclude this section, it was observed that Matj9999 produced data volumes that were inconsistent with the best preforming arcs due to editing during the orbit estimation procedures and thus the minor improvement. However, Matj9999 provided valuable data in arcs where the data was limited which improved the geocentre uncertainties for these arcs. This is the advantage of adding another station in the network. The next section presents results were the SLR station at Hartebeesthoek was removed from the estimation process. The aim of this analysis is to identify the influence this removal would have on the estimated uncertainties.
5.3 SOLUTION #3: BASELINE DATA + SIMULATED DATA, HARTRAO REMOVED FROM NETWORK

Ideally, removing observations from the network is not suitable as this limits the data quantity. However, the aim of this set of results is to further investigate the influence of this network configuration change in the previously estimated uncertainties of the SIM_SOL and the DYNAMIC_SOL solutions. During 2011, the MOBLAS-6 (Hart7501) SLR station had 20 arcs worth of observations between 03 March 2011 and 20 November 2011. Table 7 below summarises the results of this estimation process in terms of the mean and RMS of the geocentre estimates.

Table 7: Mean and RMS of the geocentre estimates from the final configuration

<table>
<thead>
<tr>
<th></th>
<th>Xcom (mm)</th>
<th>Ycom (mm)</th>
<th>Zcom (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN</td>
<td>(0.7)</td>
<td>(3.0)</td>
<td>(1.0)</td>
</tr>
<tr>
<td>RMS</td>
<td>(3.3)</td>
<td>(4.2)</td>
<td>(10.4)</td>
</tr>
</tbody>
</table>

Compared to the baseline solution in section 5.1, the estimates depicted in Table 7 agree to within 0.6, 1.3 and 0.4 mm in the X, Y and Z directions respectively. This represents the shifts emanating from the change in the network configuration. The estimates in Table 7 are also compared to the solution obtained when the simulated data was added, the SIM_SOL solution. The reader should keep in mind that these solutions assume that Matj9999 is the only SLR station in Southern Africa. Compared to the SIM_SOL estimates, the estimates in Table 7 agree to within 0.1, 0.4 and 0.1 mm in the X, Y and Z directions indicating sub millimetre shifts from the SIM_SOL estimates in X, Y and Z components. The RMS values in Table 7 agree to within 0.3, 0.9 and 1.2 mm in X, Y and Z directions when compared to the DYNAMIC_SOL, and agree to within 0.1, 0.1 and 1.9 mm in X, Y and Z directions when compared to SIM_SOL. It is clear from these values that the distribution of these estimates is more similar to the SIM_SOL solution than the DYNAMIC_SOL solution. This distribution is depicted in Figure 16 below together with weekly uncertainty estimates.
As can be seen from Figure 16 above, the removal of Hart7501 affected largely the arc of 17 Jul 2011, depicted as large peaks of 10 and 40 mm clearly visible in the plots of the Y and Z components. The reason for this arc poorly performing with respect to other arcs is that the observation quantity in this date for all satellites was low. ETALON-1 contributed 67 observations and ETALON-2 contributed 123 observations. Both LAGEOS-1 and LAGEOS-2 contributed 628 and 815 observation respectively. These quantities include the simulated station Matj9999 in them. This meant that only 1633 observations equations were processed in NASA SOLVE (Ullman, 1996) for this arc which is usually contributed by one LAGEOS satellite. The estimated uncertainties showed an increase in the same arc.

To assess the spread of these estimates, the statistical ranges for the X, Y and Z geocentre estimates were determined to be 17.2, 18.4 and 70.4 mm respectively. This shows a wider spread in all components when compared to the SIM_SOL solution especially in the Z component which is 20.0 mm wider than the SIM_SOL Z components. As the main quality indicator, the mean of uncertainties in X, Y and Z components were estimated to be 0.7, 0.7 and 2.0 mm respectively. These estimates show an increase of 0.1 mm when compared to the DYNAMIC_SOL Z component and an increase of 0.2 mm when compared to the SIM_SOL Z component. These values show consistency with both the DYNAMIC_SOL and SIM_SOL
solutions. The RMS of these uncertainties was determined to be 0.7, 0.7 and 2.2 mm in the X, Y and Z directions.

It turns out that removing Hart7501 doesn’t have a drastic influence on the accuracies of estimating the dynamic geocentre as the obtainable accuracies degrade by only 0.1 mm in the X – Y plane and 0.2 mm in the Z. This means that moving all operations to Matjiesfontein will not deteriorate the accuracies of the dynamic geocentre estimates so long as there is an operational station in Southern Africa. This will also reduce maintenance costs for the SLR system housed at HartRAO. However, having two stations operational in the country will positively affect the accuracies of estimating the geocentre.
6. CONCLUSION

As part of upgrading its space geodesy operations, the Hartebeesthoek Space Geodesy Program (SA) is planning to relocate its activities to a new site which is environmentally favourable and assist in optimising data quantity for SLR, GPS, VLBI and DORIS in order to improve products contributed to these techniques. Matjiesfontein in the Western Cape is one of the prospective locations amongst Lesotho and Sutherland in South Africa. This study focuses on the possible contribution that the SLR station to be hosted at Matjiesfontein will have, should it operate within the global network. This contribution refers directly to the determination of SLR’s unique parameter, the geocentre dynamically by estimating the degree 1 spherical harmonic coefficients of the Earth’s gravity field as utilising a constrained core network solution. This is the dynamic geocentre.

As the SLR station doesn’t exist yet, this assessment was performed via a simulation of the range observations for Matjiesfontein (Matj9999) utilising satellite orbit solutions obtained from the baseline solutions, and GPS coordinates of the station proposed to be at Matjiesfontein. The study utilises range observation to the 4 geodetic satellites LAGEOS-1, LAGEOS-2, ETALON-1 and ETALON-2. The duration of the study is from 02 Jan 2011 to 25 Dec 2011. The contribution of this simulated station refers to its improvement in the uncertainties of the dynamic geocentre estimates.

To assess this station’s contribution, three different combination procedures were proposed. The first was to process the original network and estimate the dynamic geocentre estimates together with their uncertainties in weekly arcs. This was termed DYNAMIC_SOL solution. The second solution included the simulated data in the together with the current existing global network, termed SIM_SOL solution. The third solution involved the removal of the SLR station (Hart7501) from the SIM_SOL solution.

The mean and RMS estimates for those uncertainties determined from the baseline solution (DYNAMIC_SOL) were determined to be 0.7 and 0.7 mm, 0.6 and 0.6 mm, and 1.9 and 1.9 mm for X, Y and Z components. The mean and RMS of the geocentre coordinates from this solutions were observed to be -0.1 and 3.6 mm, -4.3 and 5.1 mm, and -0.6 and 9.2 mm. From the SIM_SOL which included the simulated data, the mean and RMS estimates of uncertainties were 0.6 and 0.6 mm, 0.6 and 0.6 mm, and 1.8 and 1.8 mm for the X, Y and Z components. The third combination strategy, in which Hart7501 was removed from the
network, yielded the mean and RMS values of 0.7 and 0.7 mm, 0.7 and 0.7 mm, and 2.0 and 2.2 mm in X, Y and Z components.

From the mean uncertainties, the SIM_SOL solution led to an improvement of only 0.1 mm in all components whereas the estimates obtained when Hart7501 was remove led to a 0.1 mm increase from the baseline solution. It was identified that the minor improvement in uncertainty estimates for the simulated data emanated from the removal of the simulated data in some arcs during editing in the GEODYN II software.

The greatest challenge was ensuring that Matj9999 (Matjiesfontein) maintains a similar data quantity compared to the best performing stations in the ILRS network, namely Zimmerswald and Yaragadee. This was the main reason that the noise level assigned for Matjiesfontein was varied until a sufficient amount of data passed the editing stage. The process of selecting the noise level was experimental and didn’t involve any empirical data for the factors that would contribute towards the noise such atmospheric refraction. In reality, the SLR station at Matjiesfontein would have to yield more data to improve the geocentre estimates due to a strengthened network.
7. RECOMMENDATIONS

Based on the conclusion in the previous chapter, the following recommendations are made:

1. Empirical data such as atmospheric data should be utilised when selecting a reasonable noise level for the simulated station.
2. The time series utilised in this study is relatively short to observe any noticeable periodic and seasonal trends in geocentre estimates. For a longer time series of estimates, the results obtained in this study would differ due to the inclusion of more data and the improvement in accuracies would be more prominent.
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